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Decuplet contribution to the meson–baryon scattering lengths

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Abstract. We calculate decuplet contributions to the s-wave pseudoscalar meson octet–baryon scattering lengths to the third order in heavy baryon chiral perturbation theory (HBχPT). Using experimental pion–nucleon and kaon–nucleon scattering lengths as inputs, we determine low-energy constants and predict other meson–baryon scattering lengths. Numerically we consider three cases: (1) the case with only baryon octet contributions; (2) with decuplet contributions and (3) in the large N_c limit. Hopefully, the analytical expressions and the predictions are helpful to future investigations of the meson–baryon scattering lengths.

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1 Introduction

Chiral perturbation theory involving only pseudoscalar mesons is expanded with p/Λ_χ where p represents the meson mass or momentum, and $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking. When ground baryons are incorporated in the Lagrangian, the chiral expansion is problematic because of terms like $M_0/\Lambda_\chi \sim 1$, where M_0 is the baryon mass in the chiral limit. This problem is overcome in HBχPT [1–3] by going to extremely non-relativistic limit. Now one makes the dual expansion of p/Λ_χ and p/M_0 simultaneously, where p also represents the small residue momentum of baryons in the non-relativistic limit.

In low energy processes, decuplet baryon contributions may be important. Firstly, the mass difference between decuplet and octet baryons $\delta = 294$ MeV is not large. Furthermore, this value vanishes in the large N_c limit [4, 5]. Secondly, the coupling constant of decuplet and octet baryons with pseudoscalar mesons is large. Thus, the inclusion of these states may cancel some intermediate octet contributions. In fact, decuplet contributions partially cancels the large octet contribution in baryon axial currents [2].

One of the systematic approaches to include decuplet baryons in HBχPT is the small scale expansion (SSE)[6]. Within this counting scheme, the meson masses, all the momenta and δ are all of order $\mathcal{O}(\epsilon)$. This formalism was widely used to study processes involving explicit $J = \frac{3}{2}$ fields [7–14]. Besides SSE, an alternative counting scheme was proposed in [15–17].

For the elastic scattering of the pseudoscalar meson and octet–baryon, the scattering length a_{PB} is an important observable, which is related to the threshold T-matrix by $T_{\mathrm{PB}} = 4\pi(1 + \frac{m_{\mathrm{P}}}{M_B})a_{\mathrm{PB}}$. HB$\chi$PT provides a model-independent approach to calculate this threshold parameter. Chiral corrections to pion–nucleon scattering lengths were first investigated in two-flavor HBχPT in [18, 19]. Intermediate Δ corrections to them can be found in [10].

For the other meson–baryon interactions, one has to work in the SU(3) framework. Now the convergence of the chiral expansion has to be investigated channel by channel because of the large mass m_{K} or m_η. In [20], the s-wave kaon–nucleon scattering lengths were calculated to $\mathcal{O}(p^3)$ in SU(3) HBχPT. We calculated chiral corrections to octet-meson octet–baryon scattering lengths to the third order [21]. In the present work, we will consider the decuplet baryon contributions to the threshold meson–baryon amplitudes to $\mathcal{O}(\epsilon^3)$ in SSE in SU(3) HBχPT.

In the previous calculations [20, 21], the counter-term contributions at $\mathcal{O}(p^3)$ were assumed to be much smaller than the loop contributions. This rather naive assumption is an extension of the SU(2) case [18] where the counter-terms were estimated with resonance saturation method and found to be small. The assumption was used partly because the complete third order meson–baryon chiral Largrangians were unknown. Recently, the complete and minimal Lorentz invariant SU(3) chiral Lagrangians were composed to $\mathcal{O}(p^3)$ [22–24]. One needs to consider the counter-term contributions now.

In the following section, we collect the basic definitions and Lagrangians. We present decuplet contributions to the threshold T-matrices in Sect. 3 and the counter-terms for the third-order T-matrices Sect. 4. Then we determine the low-energy constants (LECs) in Sect. 5 by considering the counter-term contributions. The final section is our numerical results and discussions.

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2 Lagrangians

The Lagrangian of HBχPT with octet baryons has the form

$$\mathcal{L} = \mathcal{L}_{\phi\phi} + \mathcal{L}_{\phi B}\,, \tag{1}$$

where ϕ represents the pseudoscalar meson octet and B represents the baryon octet. The purely mesonic part $\mathcal{L}_{\phi\phi}$ incorporates even chiral order terms while $\mathcal{L}_{\phi B}$ starts from $\mathcal{O}(p)$. When decuplet baryons are incorporated into the system, an additional part $\mathcal{L}_{\phi \mathrm{BT}}$ is introduced in (1) where T represents the baryon decuplet. The lowest order Lagrangians of the three parts are

$$\mathcal{L}^{(2)}_{\phi\phi} = f^2 \mathrm{tr}\left(u_\mu u^\mu + \frac{\chi_+}{4}\right), \tag{2}$$

$$\mathcal{L}^{(1)}_{\phi B} = \mathrm{tr}\left(\overline{B}(\mathrm{i}\partial_0 B + [\Gamma_0, B])\right) - D\mathrm{tr}\left(\overline{B}\{\boldsymbol{\sigma}\cdot\mathbf{u}, B\}\right) - F\mathrm{tr}\left(\overline{B}[\boldsymbol{\sigma}\cdot\mathbf{u}, B]\right), \tag{3}$$

$$\mathcal{L}^{(1)}_{\phi\mathrm{BT}} = -\overline{T}^\mu\left(\mathrm{i}\mathcal{D}_0 - \delta\right)T_\mu + \mathcal{C}\left(\overline{T}^\mu u_\mu B + \overline{B} u_\mu T^\mu\right) - \mathcal{H}\overline{T}^\mu \boldsymbol{\sigma}\cdot\mathbf{u} T_\mu\,, \tag{4}$$

where δ is the decuplet and octet baryon mass difference in the chiral limit and the common notations read

$$\Gamma_\mu = \frac{\mathrm{i}}{2}\left[\xi^\dagger, \partial_\mu \xi\right], \quad u_\mu = \frac{\mathrm{i}}{2}\left\{\xi^\dagger, \partial_\mu \xi\right\}, \quad \xi = \exp(\mathrm{i}\phi/2f)\,, \tag{5}$$

$$\chi_\pm = \xi^\dagger \chi \xi^\dagger \pm \xi \chi \xi\,, \quad \chi = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_\mathrm{K}^2 - m_\pi^2)\,, \tag{6}$$

$$\phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{\mathrm{K}}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \tag{7}$$

$$\mathrm{i}\mathcal{D}_\mu T^\nu_{abc} = \mathrm{i}\partial_\mu T^\nu_{abc} + (\Gamma_\mu)^d_a T^\nu_{dbc} + (\Gamma_\mu)^d_b T^\nu_{adc} + (\Gamma_\mu)^d_c T^\nu_{abd}\,, \tag{8}$$

and

$$T_{111} = \Delta^{++}, \quad T_{112} = \frac{\Delta^+}{\sqrt{3}}, \quad T_{122} = \frac{\Delta^0}{\sqrt{3}}, \quad T_{222} = \Delta^-,$$
$$T_{113} = \frac{\Sigma^{*+}}{\sqrt{3}}, \quad T_{123} = \frac{\Sigma^{*0}}{\sqrt{6}}, \quad T_{223} = \frac{\Sigma^{*-}}{\sqrt{3}},$$
$$T_{133} = \frac{\Xi^{*0}}{\sqrt{3}}, \quad T_{233} = \frac{\Xi^{*-}}{\sqrt{3}}, \quad T_{333} = \Omega^-\,. \tag{9}$$

f is the pseudoscalar meson decay constant in the chiral limit. Γ_μ is the chiral connection which contains even numbers of meson fields. u_μ contains odd numbers of meson fields. $D + F = g_\mathrm{A} = 1.26$ where g_A is the axial vector coupling constant. The superscripts in these Lagrangians represent the order of the small scale expansion.

In our calculation of decuplet contributions to threshold pseudoscalar meson octet–baryon scattering T-matrices, we truncate at $\mathcal{O}(\epsilon^3)$. In this case, $\mathcal{L}^{(2)}_{\phi B}$ does not contribute, which can be found in [20, 21]. Similarly, $\mathcal{L}^{(2)}_{\phi\mathrm{BT}}$ or high order Lagrangians also have vanishing contributions.

Recently, the complete three-flavor Lorentz-invariant meson–baryon chiral Lagrangians have been composed to the third order [22–24]. Only three independent terms will contribute to the meson–baryon scattering T-matrices at threshold

$$\mathcal{L}^{(3)}_{\phi B} = h_1 \mathrm{tr}(\overline{B}B[\chi_-, u_0]) + h_2 \mathrm{tr}(\overline{B}[\chi_-, u_0]B) + h_3\{\mathrm{tr}(\overline{B}u_0)\mathrm{tr}(\chi_- B) - \mathrm{tr}(\overline{B}\chi_-)\mathrm{tr}(u_0 B)\}\,, \tag{10}$$

where h_1, h_2 and h_3 are LECs, which also play the role of absorbing divergences from loop calculations. When transforming the relativistic Lagrangian into the heavy baryon formalism, additional $1/M_0$ corrections may in principle appear. However, these kinds of recoil corrections are higher than our truncation order. Thus the above Lagrangian may also be treated as the form in HBχPT.

3 Decuplet contributions to threshold T-matrices

There are many diagrams for a general elastic pseudoscalar meson octet–baryon scattering process to the third chiral order. When intermediate decuplet contributions are considered, there are additional diagrams. However, the calculation is simpler at threshold. One may consult [20, 21] for the case with only intermediate octet baryon. Here we consider intermediate decuplet contributions. Decuplet corrections at the tree level vanish either due to $\boldsymbol{\sigma}\cdot\mathbf{q} = 0$ or $q^\mu P^{3/2}_{\mu\nu} = 0$. Here $\boldsymbol{\sigma}$ is the Pauli spin vector, q is the momentum of the external meson and $P^{3/2}_{\mu\nu}$ is the projection operator of Rarita–Schwinger field. In the d-dimension space, $P^{3/2}_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu + \frac{4}{d-1} S_\mu S_\nu$ where $g_{\mu\nu}$ is the metric tensor, v_μ is a four-velocity and S_μ is the Pauli–Lubanski spin vector. Corrections to the one-loop diagrams start from $\mathcal{O}(\epsilon^3)$ in the small scale expansion. There are six non-vanishing diagrams at this order, which we show in Fig. 1. The vertices in the figure are generated from $\mathcal{L}^{(1)}_{\phi B}$, $\mathcal{L}^{(1)}_{\phi\mathrm{BT}}$ and $\mathcal{L}^{(2)}_{\phi\phi}$.

In the previous loop calculations [20, 21], dimensional regularization and minimal subtraction were used. The divergences were completely absorbed by the LECs in $\mathcal{L}^{(3)}_{\phi B}$. When we consider the additional diagrams due to intermediate baryon decuplet, there are divergence that the LECs will not absorb. We give the finite parts from the loop calculation in this section and give the counter-terms in the next section.

To write down the threshold T-matrices in compact forms, we define

$$W(m^2) = \begin{cases} \delta \ln\frac{|m|}{\lambda} + \sqrt{\delta^2 - m^2}\ln\frac{\delta + \sqrt{\delta^2 - m^2}}{|m|}\,, & \text{if } (m^2 < \delta^2) \\ \delta \ln\frac{|m|}{\lambda} - \sqrt{m^2 - \delta^2}\arccos\frac{\delta}{|m|}\,, & \text{if } (m^2 > \delta^2) \end{cases} \tag{11}$$

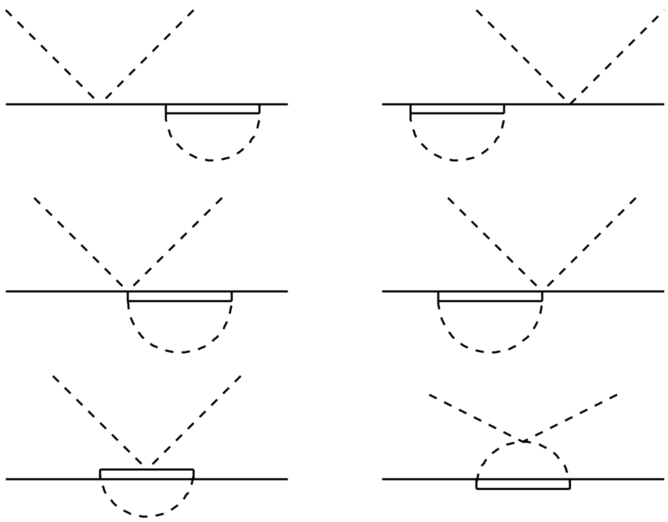

Fig. 1. Non-vanishing diagrams for meson–baryon scattering with intermediate decuplet contributions at threshold. *Dashed lines* represent Goldstone bosons, *full lines* represent octet baryons, and *double lines* represent decuplet baryons

where m represents the meson mass and λ is the scale from dimensional regularization. We list the T-matrices below. For πN scattering, we have

$$T_{\pi N}^{(3/2)} = T_{\pi N}^{(1/2)} = \frac{\mathcal{C}^2}{12\pi^2 f_\pi^4}\left\{-m_\pi^2 W\left(m_\pi^2\right)\right\}, \quad (12)$$

or

$$T_{\pi N}^{+} = -\frac{\mathcal{C}^2 m_\pi^2}{12\pi^2 f_\pi^4}\left\{W\left(m_\pi^2\right)\right\},$$
$$T_{\pi N}^{-} = 0. \quad (13)$$

For $\pi\Sigma$ and $\pi\Xi$ scattering T-matrices,

$$T_{\pi B}^{(I)} = \frac{\mathcal{C}^2 m_\pi}{48\pi^2 f_\pi^4}\left\{m_\pi\left[\zeta_B^{(I)} W\left(m_\pi^2\right) + W\left(m_\eta^2\right)\right]\right\}, \quad (14)$$

where I labels the total isospin and B the baryon Σ or Ξ,

$$\zeta_\Sigma^{(2)} = -1\,, \quad \zeta_\Sigma^{(1)} = 1\,, \quad \zeta_\Sigma^{(0)} = -4\,, \quad (15)$$

and

$$\zeta_\Xi^{(3/2)} = -1\,; \quad \zeta_\Xi^{(1/2)} = -1\,. \quad (16)$$

The T-matrix for $\pi\Lambda$ scattering is very simple,

$$T_{\pi\Lambda} = -\frac{\mathcal{C}^2 m_\pi^2}{16\pi^2 f_\pi^4}\left\{W\left(m_\pi^2\right)\right\}. \quad (17)$$

The kaon–nucleon scattering T-matrices are

$$T_{MN}^{(I)} = 0, \quad (18)$$

where M represents K or $\overline{K}$ and $I = 1$ or $I = 0$.

The T-matrices for kaon-Σ and kaon-Ξ scatterings are complicated due to the last diagram in Fig. 1 with an intermediate η and an intermediate π^0. If we define

$$J = \frac{1}{3}\delta\left(1 - 6\ln\frac{m_\eta}{\lambda}\right) - \frac{2\delta\left(3m_\pi^2 - 2\delta^2\right)}{3\left(m_\eta^2 - m_\pi^2\right)}\ln\frac{m_\eta}{m_\pi} + \frac{4}{3\left(m_\eta^2 - m_\pi^2\right)}\left[\left(m_\eta^2 - \delta^2\right)^{\frac{3}{2}}\arccos\frac{\delta}{m_\eta} - \left(\delta^2 - m_\pi^2\right)^{\frac{3}{2}}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}\right], \quad (19)$$

the matrices for these two channels read:

$$T_{MB}^{(I)} = \frac{\mathcal{C}^2 m_K}{48\pi^2 f_K^4}\left\{m_K\left[\zeta_{MB}^{(I)} J - 2W\left(m_\eta^2\right)\right]\right\}, \quad (20)$$

where M represents K or $\overline{K}$, B represents Σ or Ξ,

$$\zeta_{K\Sigma}^{(3/2)} = 1\,, \quad \zeta_{K\Sigma}^{(3/2)} = -2\,, \quad \zeta_{\overline{K}\Sigma}^{(3/2)} = -1\,, \quad \zeta_{\overline{K}\Sigma}^{(3/2)} = 2\,; \quad (21)$$

and

$$\zeta_{K\Xi}^{(1)} = 1\,, \quad \zeta_{K\Xi}^{(0)} = -3\,, \quad \zeta_{\overline{K}\Xi}^{(1)} = -1\,, \quad \zeta_{\overline{K}\Xi}^{(0)} = 1\,. \quad (22)$$

The T-matrices for $T_{\overline{K}\Lambda}$ and $T_{K\Lambda}$ vanish.

The four ηB T-matrices depend on $W\left(m_\pi^2\right)$, $W\left(m_K^2\right)$ and $W\left(m_\eta^2\right)$.

$$T_{\eta B} = \frac{\mathcal{C}^2}{144\pi^2 f_\eta^4}\left\{\alpha_B m_\pi^2 W\left(m_\pi^2\right) + \beta_B m_K^2 W\left(m_K^2\right) + \gamma_B (4m_\eta^2 - m_\pi^2) W\left(m_\eta^2\right)\right\}, \quad (23)$$

where

$$\alpha_N = 12\,, \quad \beta_N = -6\,, \quad \gamma_N = 0; \quad \alpha_\Sigma = 2\,, \quad \beta_\Sigma = -20\,, \quad \gamma_\Sigma = 3\,; \quad \alpha_\Xi = 3\,, \quad \beta_\Xi = -18\,, \quad \gamma_\Xi = 3; \quad \alpha_\Lambda = 9\,, \quad \beta_\Lambda = -12\,, \quad \gamma_\Lambda = 0\,. \quad (24)$$

In these loop expressions, we have used f_π in pion processes, f_K in kaon processes and f_η in eta processes. Different usage of decay constants leads to deviations at higher order. Therefore the difference in the numerical results is expected to be negligible.

From these T-matrices, we see the intermediate decuplet states do not generate any corrections for the imaginary part of the threshold T-matrices. One can verify that the kaon–baryon and anti-kaon–baryon T-matrices satisfy the crossing symmetry [20, 21]. In the SU(3) limit, the relations in [21] also hold. However, the similarity for T-matrices involving isospin doublets do not exist any longer.

4 Counter-terms

In the previous calculation of the threshold T-matrices, the contributions from the renormalized counter-terms were naively assumed to be much smaller than chiral loop corrections and ignored in the numerical analysis. With the complete third-order Lagrangian $\mathcal{L}_{\phi B}^{(3)}$ [22–24], we need to include the counter-terms explicitly at $\mathcal{O}(\epsilon^3)$ from (10).

$$T_{\pi N}^{(3/2)} = -4h_2^{\rm r}\frac{m_\pi^3}{f_\pi^2}\,, \quad T_{\pi N}^{(1/2)} = 8h_2^{\rm r}\frac{m_\pi^3}{f_\pi^2}\,, \quad T_{\pi N}^{+} = 0\,,$$
$$T_{\pi N}^{-} = 4h_2^{\rm r}\frac{m_\pi^3}{f_\pi^2}\,; \tag{25}$$
$$T_{\pi\Sigma}^{(2)} = 4(h_1^{\rm r}-h_2^{\rm r}+h_3)\frac{m_\pi^3}{f_\pi^2}\,,$$
$$T_{\pi\Sigma}^{(1)} = -4(h_1^{\rm r}-h_2^{\rm r}+h_3)\frac{m_\pi^3}{f_\pi^2}\,,$$
$$T_{\pi\Sigma}^{(0)} = -8(h_1^{\rm r}-h_2^{\rm r}+h_3)\frac{m_\pi^3}{f_\pi^2}\,; \tag{26}$$
$$T_{\pi\Xi}^{(3/2)} = 4h_1^{\rm r}\frac{m_\pi^3}{f_\pi^2}\,, \quad T_{\pi\Xi}^{(1/2)} = -8h_1^{\rm r}\frac{m_\pi^3}{f_\pi^2}\,; \tag{27}$$
$$T_{{\rm K}N}^{(1)} = 4(h_1^{\rm r}-h_2^{\rm r}+h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,,$$
$$T_{{\rm K}N}^{(0)} = 4(h_1^{\rm r}+h_2^{\rm r}-h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,,$$
$$T_{\overline{\rm K}N}^{(1)} = -4h_1^{\rm r}\frac{m_{\rm K}^3}{f_{\rm K}^2}\,, \quad T_{\overline{\rm K}N}^{(0)} = -4(h_1^{\rm r}-2h_2^{\rm r}+2h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,; \tag{28}$$
$$T_{{\rm K}\Sigma}^{(3/2)} = -4h_2^{\rm r}\frac{m_{\rm K}^3}{f_{\rm K}^2}\,, \quad T_{{\rm K}\Sigma}^{(1/2)} = -2(3h_1^{\rm r}-h_2^{\rm r})\frac{m_{\rm K}^3}{f_{\rm K}^2}\,,$$
$$T_{\overline{\rm K}\Sigma}^{(3/2)} = 4h_1^{\rm r}\frac{m_{\rm K}^3}{f_{\rm K}^2}\,, \quad T_{\overline{\rm K}\Sigma}^{(1/2)} = -2(h_1^{\rm r}-3h_2^{\rm r})\frac{m_{\rm K}^3}{f_{\rm K}^2}\,; \tag{29}$$
$$T_{{\rm K}\Xi}^{(1)} = 4h_2^{\rm r}\frac{m_{\rm K}^3}{f_{\rm K}^2}\,, \quad T_{{\rm K}\Xi}^{(0)} = -4(2h_1^{\rm r}-h_2^{\rm r}+2h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,,$$
$$T_{\overline{\rm K}\Xi}^{(1)} = 4(h_1^{\rm r}-h_2^{\rm r}+h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,,$$
$$T_{\overline{\rm K}\Xi}^{(0)} = -4(h_1^{\rm r}+h_2^{\rm r}+h_3)\frac{m_{\rm K}^3}{f_{\rm K}^2}\,; \tag{30}$$
$$T_{\overline{\rm K}\Lambda} = -T_{{\rm K}\Lambda} = -2(h_1^{\rm r}+h_2^{\rm r})\frac{m_{\rm K}^3}{f_{\rm K}^2}\,; \tag{31}$$
$$T_{\pi\Lambda} = T_{\eta N} = T_{\eta\Sigma} = T_{\eta\Xi} = T_{\eta\Lambda} = 0\,. \tag{32}$$

Among these counter-terms, $h_1^{\rm r}$ and $h_2^{\rm r}$ are renormalized LECs while h_3 is finite. The renormalized LECs connect with unrenormalized ones through $h_1^{\rm r} = h_1 + \frac{3}{4}\frac{L}{f^2}$ and $h_2^{\rm r} = h_2 - \frac{3}{4}\frac{L}{f^2}$ when decuplet was not included. $L = \frac{\lambda^{d-4}}{16\pi^2}[\frac{1}{d-4} + \frac{1}{2}(\gamma_{\rm E} - 1 - \ln 4\pi)]$ where d is the space-time dimension and $\gamma_{\rm E}$ is the Euler constant. When decuplet contributions are considered, loops generate divergences proportional to $\delta m^2 \frac{\mathcal{C}^2 L}{f^4}$, which can not be candelled. This is the result of incompleteness for the renormalization in HBχPT with $J = \frac{3}{2}$ states because of non-vanishing δ. One notes this term is of $\frac{1}{f^4}$ and analytical both in δ and m. In the large $N_{\rm c}$ limit, $\delta \to 0$ [4,5], the correspondence between divergences and counter-terms recovers. One practical approach is to keep non-analytical chiral corrections only in the numerical analysis and throw away the above divergent term with analytical coefficients.

One notes that the LECs $h_1^{\rm r}$ and $h_2^{\rm r}$ are in fact scale-dependent. They cancel the scale-dependent parts arising from loops, which makes the resulting T-matrices scale-independent. One may define scale-independent LECs when only baryon octet is involved. However, when decuplet is considered, one can not find self-consistent definitions of scale-independent LECs. One can verify these counter-terms satisfy the crossing symmetry and SU(3) limit relations in [21].

5 Low-energy constants

To calculate the scattering lengths numerically, one has to determine the LECs and their combinations. There are eight and three parameters in the second and third-order Lagrangians, respectively. The final T-matrices to $\mathcal{O}(\epsilon^3)$ involve five LECs b_D, b_F, $h_1^{\rm r}$, $h_2^{\rm r}$, h_3 and four LEC combinations $C_{1,0,\pi,d}$ which were defined as [20, 21]

$$C_1 = 2(d_0-2b_0)+2(d_D-2b_D)+d_1-\frac{D^2+3F^2}{6M_0}\,,$$
$$C_0 = 2(d_0-2b_0)-2(d_F-2b_F)-d_1-\frac{D(D-3F)}{3M_0}\,,$$
$$C_\pi = (d_F-2b_F)-\frac{DF}{2M_0}\,, \quad C_d = d_1-\frac{D^2-3F^2}{6M_0}\,, \tag{33}$$

where b_0, b_D, b_F, d_0, d_D, d_F and d_1 come from $\mathcal{L}_{\phi B}^{(2)}$. It is impossible to combine the LECs $h_{1,2,3}$ with $C_{1,0,\pi,d}$ to reduce the number of parameters. Up to now, we can determine most of them with available inputs. Unfortunately, one is unable to extract C_d strictly from known sources. We simply estimate its value.

In the SU(2) case, numerical evaluations for observables with either scale-dependent or scale-independent LECs give the same results. In the SU(3) case, symmetry is largely broken. The two usages of LECs may result in some deviations for current T-matrices, especially when baryon decuplet contributions are considered. To reduce the effects from symmetry breaking, we use scale-dependent $h_1^{\rm r}$ and $h_2^{\rm r}$ in the following calculations. This choice is also convenient for the discussion about the assumption used in [20, 21].

When determining the LECs, our procedure is as follows. (i) We first choose the scale $\lambda = 4\pi f_\pi$, which is widely used in chiral perturbation theory. With the six pion–nucleon and kaon–nucleon scattering lengths we determine $C_{1,0,\pi}$ and $h_1^{\rm r}, h_2^{\rm r}, h_3$; (ii) then we determine M_0, b_0, b_D, b_F by fitting the baryon masses and $\sigma_{\pi N}$; (iii) finally, we use these parameters as inputs to estimate other LECs and C_d. The errors will also be estimated with the error propagation formula. We consider three cases: the case with only baryon octet contributions, with decuplet contributions and in the large $N_{\rm c}$ limit.

Few experimental scattering lengths are available. Recently, those for πN scattering were measured [25]: $a^+_{\pi N} = -0.0001^{+0.0009}_{-0.0021}\, m_\pi^{-1}$, $a^-_{\pi N} = 0.0885^{+0.0021}_{-0.0010}\, m_\pi^{-1}$. The new datum for a_{K^-p} [26] is not enough for our purpose. We use the empirical values for kaon–nucleon scattering lengths from [27],

$$a^{(1)}_{\mathrm{K}N} = -0.33\,\mathrm{fm}\,,\quad a^{(0)}_{\mathrm{K}N} = 0.02\,\mathrm{fm}\,,$$
$$a^{(1)}_{\overline{\mathrm{K}}N} = 0.37 + 0.60\mathrm{i}\,\mathrm{fm}\,,\quad a^{(0)}_{\overline{\mathrm{K}}N} = -1.70 + 0.68\mathrm{i}\,\mathrm{fm}. \tag{34}$$

For the parameters in the expressions of T-matrices, we use [28]

$$\begin{aligned} &m_\pi = 139.57\,\mathrm{MeV}\,,\quad m_\mathrm{K} = 493.68\,\mathrm{MeV}\,,\\ &m_\eta = 547.75\,\mathrm{MeV}\,,\\ &\delta = 294\,\mathrm{MeV}\,,\quad f_\pi = 92.4\,\mathrm{MeV}\,,\quad f_\mathrm{K} = 113\,\mathrm{MeV}\,,\\ &f_\eta = 1.2 f_\mathrm{K}\\ &D = 0.75\,,\quad F = 0.5\,,\quad \mathcal{C} = -1.5\,. \end{aligned} \tag{35}$$

Now we reconsider the case with only baryon octet contributions by including the counter-terms. The loop expressions can be found in [21]. When we express $C_{1,0,\pi}$, h^r_1, h^r_2 and h_3 with $a^+_{\pi N}$, $a^-_{\pi N}$, $a^{(1)}_{\mathrm{K}N}$, $a^{(0)}_{\mathrm{K}N}$, $\mathrm{Re}[a^{(1)}_{\overline{\mathrm{K}}N}]$ and $\mathrm{Re}[a^{(0)}_{\overline{\mathrm{K}}N}]$, we get

$$\begin{aligned} &C_1 = -2.339\,\mathrm{GeV}^{-1}\,,\quad C_0 = 4.389\,\mathrm{GeV}^{-1}\,,\\ &C_\pi = 0.152^{+0.020}_{-0.048}\,\mathrm{GeV}^{-1}\,,\\ &h^\mathrm{r}_1 = 0.037\,\mathrm{GeV}^{-2}\,,\quad h^\mathrm{r}_2 = -0.274^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,,\\ &h_3 = 1.769^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,. \end{aligned} \tag{36}$$

With the mass formulas in [3,29], we get b_D, b_F, b_0 and M_0 by fitting baryon masses $M_N = 938.9 \pm 1.3\,\mathrm{MeV}$, $M_\Sigma = 1193.4 \pm 8.1\,\mathrm{MeV}$, $M_\Xi = 1318.1 \pm 6.7\,\mathrm{MeV}$, $M_\Lambda = 1115.7 \pm 5.4\,\mathrm{MeV}$ and $\sigma_{\pi N} = 45 \pm 8\,\mathrm{MeV}$ [30],

$$\begin{aligned} &M_0 = 808.94 \pm 104.20\,\mathrm{MeV}\,,\quad b_0 = -0.786 \pm 0.103\,\mathrm{GeV}^{-1}\,,\\ &b_D = 0.028 \pm 0.008\,\mathrm{GeV}^{-1}\,,\quad b_F = -0.473 \pm 0.003\,\mathrm{GeV}^{-1}\,, \end{aligned} \tag{37}$$

with $\chi^2/\mathrm{d.o.f.} \simeq 0.75$. In the fitting procedure, we have used $f = f_\pi$ in π loops, $f = f_\mathrm{K}$ in kaon loops and $f = f_\eta$ in η loops in the formulas. The results differ slightly from our previous values only, because we used a smaller D.

From the above determined quantities, we deduce $d_F = -0.562^{+0.037}_{-0.057}\,\mathrm{GeV}^{-1}$ with $d_F = C_\pi + 2b_F + \frac{DF}{2M_0}$. Similarly, if we use the second order $d_0 = -0.996\,\mathrm{GeV}^{-1}$ [31], we have

$$\begin{aligned} &d_D = 0.331^{+0.413}_{-0.415}\,\mathrm{GeV}^{-1}\,,\quad d_1 = -3.772^{+0.414}_{-0.423}\,\mathrm{GeV}^{-1}\,,\\ &C_d = -3.733^{+0.414}_{-0.423}\,\mathrm{GeV}^{-1}\,. \end{aligned} \tag{38}$$

One notes they are estimated values because of lack of experimental inputs.

With intermediate decuplet contributions, one should note there are still divergent parts in the T-matrices which could not be absorbed. We just ignore them in the numerical evaluation as usually done. By repeating the above procedure, we get

$$\begin{aligned} &C_1 = -2.339\,\mathrm{GeV}^{-1}\,,\quad C_0 = 4.389\,\mathrm{GeV}^{-1}\,,\\ &C_\pi = -0.145^{+0.020}_{-0.048}\,\mathrm{GeV}^{-1}\,,\\ &h^\mathrm{r}_1 = 0.037\,\mathrm{GeV}^{-2}\,,\quad h^\mathrm{r}_2 = -0.274^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,,\\ &h_3 = 1.769^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,, \end{aligned} \tag{39}$$

with the updated threshold T-matrices.

When we determine b_0, b_D, b_F and M_0, decuplet corrections to baryon masses and πN sigma term have to be considered. One gets the corrections from the self energy diagram. If we define

$$I\left(m^2\right) = \left(m^2 - \delta^2\right) W\left(m^2\right) - \delta\left(\frac{1}{4}m^2 - \frac{1}{3}\delta^2\right) + \frac{1}{2}\delta m^2 \ln\frac{m}{\lambda}\,, \tag{40}$$

then the shifts of baryon masses are

$$\varDelta M_B = \frac{\mathcal{C}^2}{24\pi^2}\left\{\alpha_B \frac{I\left(m_\pi^2\right)}{f_\pi^2} + \beta_B \frac{I\left(m_\mathrm{K}^2\right)}{f_\mathrm{K}^2} + \gamma_B \frac{I\left(m_\eta^2\right)}{f_\eta^2}\right\}, \tag{41}$$

where

$$\begin{aligned} &\alpha_N = 4\,,\quad \beta_N = 1\,,\quad \gamma_N = 0;\quad \alpha_\Sigma = \frac{2}{3}\,,\quad \beta_\Sigma = \frac{10}{3}\,,\\ &\gamma_\Sigma = 1\,;\quad \alpha_\Xi = 1\,,\quad \beta_\Xi = 3\,,\quad \gamma_\Xi = 1;\quad \alpha_\Lambda = 3\,,\\ &\beta_\Lambda = 2\,,\quad \gamma_\Lambda = 0\,. \end{aligned} \tag{42}$$

The shift of πN σ term is

$$\varDelta\sigma_{\pi N} = \frac{m_\pi^2 \mathcal{C}^2}{32\pi^2}\left\{8\frac{W\left(m_\pi^2\right)}{f_\pi^2} + \frac{W\left(m_\mathrm{K}^2\right)}{f_\mathrm{K}^2}\right\}. \tag{43}$$

Here we give the finite parts only. In contrast to the case without decuplet corrections, the divergent parts are non-vanishing. Those proportional to $\delta m^2 \frac{\mathcal{C}^2 L}{f^2}$ vanish either in chiral limit or in large N_c limit. Those proportional to $\delta^3 \frac{\mathcal{C}^2 L}{f^2}$ in baryon masses vanish only in large N_c limit. In [29], a counter-term was added by hand to cancel the latter divergence. In our case, we simply ignore these regular parts. Our formulae are slightly different from those in [29].

With the updated mass and $\sigma_{\pi N}$ formulas, we get

$$\begin{aligned} &M_0 = 745.02 \pm 104.22\,\mathrm{MeV}\,,\quad b_0 = -1.342 \pm 0.103\,\mathrm{GeV}^{-1},\\ &b_D = 0.308 \pm 0.008\,\mathrm{GeV}^{-1}\,,\quad b_F = -0.705 \pm 0.003\,\mathrm{GeV}^{-1} \end{aligned} \tag{44}$$

with $\chi^2/\mathrm{d.o.f.} \simeq 0.39$, which is smaller than the former case. In this procedure, we again used f_π in pion loops, f_K in kaon loops and f_η in η loops. Further we obtain $d_F = -1.303^{+0.041}_{-0.060}\,\mathrm{GeV}^{-1}$, $d_D = -1.608^{+0.414}_{-0.416}\,\mathrm{GeV}^{-1}$, $d_1 = -0.975^{+0.415}_{-0.423}\,\mathrm{GeV}^{-1}$ and $C_d = -0.933^{+0.414}_{-0.423}\,\mathrm{GeV}^{-1}$ when we express them with $C_{1,0,\pi}$, b_0, b_D, M_0 and d_0 and use $d_0 = -0.996\,\mathrm{GeV}^{-1}$ [31] as inputs.

An important case is the large N_c limit. In this limit, $\delta = 0, \mathcal{C} = -2D$ [32]. The divergent parts related with δ in T-matrices, baryon masses and πN sigma term vanish. The defined W and J are simple:

$$W\left(m^2\right) = -\frac{1}{2}\pi|m|\,,$$
$$J = \frac{2\pi\left(m_\eta^2 + m_\pi^2 + m_\pi m_\eta\right)}{3\left(m_\eta + m_\pi\right)}\,. \qquad (45)$$

We also determine all the other LECs with $d_0 = -0.996\,\mathrm{GeV}^{-1}$ in this limit by repeating the above procedure. They are

$$\begin{aligned}
M_0 &= 808.94 \pm 104.20\,\mathrm{MeV}\,,\\
b_0 &= -0.786 \pm 0.103\,\mathrm{GeV}^{-1}\,,\\
b_D &= 0.028 \pm 0.008\,\mathrm{GeV}^{-1}\,,\\
b_F &= -0.473 \pm 0.003\,\mathrm{GeV}^{-1}\,,\\
d_F &= -0.806^{+0.037}_{-0.057}\,\mathrm{GeV}^{-1}\,,\\
d_D &= 0.087^{+0.413}_{-0.415}\,\mathrm{GeV}^{-1}\,, \quad d_1 = -3.284^{+0.414}_{-0.423}\,\mathrm{GeV}^{-1}\,,\\
h_1^{\mathrm{r}} &= -0.037\,\mathrm{GeV}^{-2}\,, \quad h_2^{\mathrm{r}} = -0.274^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,,\\
h_3 &= 1.769^{+0.171}_{-0.081}\,\mathrm{GeV}^{-2}\,. \qquad (46)
\end{aligned}$$

When we obtain the first four parameters with MINUIT, we have $\chi^2/\mathrm{d.o.f.} \simeq 0.75$. Those LEC combinations are

$$\begin{aligned}
C_1 &= -2.339\,\mathrm{GeV}^{-1}\,, \quad C_0 = 4.389\,\mathrm{GeV}^{-1}\,,\\
C_\pi &= -0.092^{+0.020}_{-0.048}\,\mathrm{GeV}^{-1}\,, \quad C_d = -3.245^{+0.414}_{-0.423}\,\mathrm{GeV}^{-1}\,. \qquad (47)
\end{aligned}$$

6 Numerical results and discussions

With the three sets of parameters, we evaluate all the meson–baryon threshold T-matrices. We present numerical results involving only octet baryons in Tables 1–3, the results for the case with decuplet contributions in Tables 4–6 and those in the large N_c limit in Tables 7–9. The corresponding scattering lengths are given in the last column. We estimate the errors from those of C_π, C_d, b_D, b_F, h_2^{r} and h_3 with the error propagation formula.

We first consider the case without explicit decuplet contributions. In this case, the contributions from the decuplet baryons, resonances close to thresholds and other baryons are all buried in the LECs.

It is interesting to see those eta–baryon scattering lengths. The widely studied η-mesic nuclei were proposed decades ago [33, 34]. The η-mesic hypernuclei were also proposed [35]. An important parameter to verify whether they exist is the eta–baryon scattering length. One can consult [21] for our previous calculation for this observable. When counter-terms were considered, we find that $\mathrm{Re}[a_{\eta\Lambda}]$ is negative, contrary to the results in [35, 36]. This is due to the large and negative contribution at the second order. From $T_{\eta\Lambda}$ at the second order and the values C_1, C_d and b_D, it is not surprising to get this result. One may think that the estimated C_d is problematic. According to $C_d = -C_0 - 2C_\pi - 4b_0 + 2d_0 - \frac{D^2-F^2}{2M_0}$, we see that even if $d_0 \sim 0.0$, C_d would still be negative. As a result, it is almost definite that $\mathrm{Re}[a_{\eta\Lambda}] < 0$.

From the results in Tables 1–3, we find that the T-matrices $T^{(1/2)}_{\pi N}$, $T^{(2)}_{\pi\Sigma}$, $T^{(1)}_{\pi\Sigma}$ and $T^{(1/2)}_{\mathrm{K}\Sigma}$ converge well. The situation differs slightly from the previous calculation [21]. Any scattering lengths related with C_d can be used as a constraint to determine LECs if they were measured, but $a^{(1)}_{\pi\Sigma}$ is particularly ideal for this purpose.

Let us discuss the contributions from counter-terms. The pion–nucleon scattering lengths were first calculated to $\mathcal{O}(p^3)$ in [18] in the SU(2) HBχPT. The counter-terms were estimated with resonance saturation method. It was found that counter-terms have much smaller contributions than chiral loop corrections. This naive assumption was extended to the SU(3) case in [20, 21]. However, such an extension is actually problematic. We have analyzed the third order T-matrices numerically. We found that counter-terms have even larger contributions than loops in the following T-matrices: $T^{(1)}_{\pi\Sigma}$, $T^{(0)}_{\pi\Sigma}$, $T^{(1)}_{\mathrm{K}N}$, $T^{(0)}_{\mathrm{K}N}$, $T^{(0)}_{\overline{\mathrm{K}}N}$, $T^{(3/2)}_{\mathrm{K}\Sigma}$, $T^{(0)}_{\mathrm{K}\Xi}$, $T^{(1)}_{\overline{\mathrm{K}}\Xi}$ and $T^{(0)}_{\overline{\mathrm{K}}\Xi}$. Fortunately, most of the predictions in [21] are not far away from the current calculation.

When the baryon decuplet contributions were considered explicitly, about half of the predictions are close to those without decuplet contribution. But several scattering lengths change sign. Now the real part of $a_{\eta N}$ has a negative sign which is contrary to results in literature (see [37] for an overview). $\mathrm{Re}(a_{\eta\Sigma}) = 0.70 \pm 0.04$ fm is still consistent with $a_{\eta\Sigma} = [(0.10 \sim 1.10) + (0.35 \sim 2.20)\mathrm{i}]$ fm in [35]. $\mathrm{Re}[a_{\eta\Lambda}]$ is still negative. The change of scattering lengths due to the inclusion of decuplet is known to be the result of non-commutativity of the chiral limit and large N_c limit. The effects of the interplay of these two limits on meson–baryon scattering lengths were recently discussed in [38].

From Tables 4–6, one finds that the chiral expansion converges for $T^{(1/2)}_{\pi N}$, $T^{(2)}_{\pi\Sigma}$, $T^{(0)}_{\pi\Sigma}$, $T^{(1/2)}_{K\Sigma}$, $T_{K\Lambda}$, and $T_{\overline{K}\Lambda}$. It was pointed out in [13, 14] that the chiral expansion in HBχPT with decuplet is worse than that in HBχPT with only ground baryons. In the present case, the convergence of scattering lenghts changes little with decuplet corrections.

All known scattering lengths have been used to determine LECs. There are no available experimental data to compare with. The present predictions require future experimental measurements to test HBχPT. On the other hand, once the four kaon–nucleon scattering lengths as well as $a^{(1)}_{\pi\Sigma}$ were measured accurately, we can predict precisely others to the third order in HBχPT. Strangeness programmes in CSR and JPARC are hopeful for these purposes.

In summary, we have calculated intermediate decuplet baryon contributions to the s-wave meson–baryon scattering lengths to the third order in small scale expansion. Hopefully, the explicit expressions are helpful to future chiral extrapolations in lattice simulations. From the known $a_{\pi N}$, $a_{\mathrm{K}N}$ and $a_{\overline{\mathrm{K}}N}$, we determined the LECs and predicted other

Table 1. Pion–baryon threshold T-matrices order by order with only octet contributions in unit of fm

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T^{+}_{\pi N}$	0	$0.60^{+0.02}_{-0.04}$	-0.60	$-0.002^{+0.018}_{-0.043}$	$-0.00014^{+0.00127}_{-0.00297}$ (input)
$T^{-}_{\pi N}$	1.61	0	$0.19^{+0.04}_{-0.02}$	$1.81^{+0.04}_{-0.02}$	$0.125^{+0.003}_{-0.001}$ (input)
$T^{(3/2)}_{\pi N}$	-1.61	$0.60^{+0.02}_{-0.04}$	$-0.79^{+0.04}_{-0.02}$	-1.81 ± 0.05	-0.130 ± 0.003
$T^{(1/2)}_{\pi N}$	3.23	$0.60^{+0.02}_{-0.04}$	$-0.21^{+0.08}_{-0.04}$	$3.61^{+0.18}_{-0.12}$	0.25 ± 0.01
$T^{(2)}_{\pi\Sigma}$	-3.23	-1.05	$-0.50^{+0.06}_{-0.03}$	$-4.77^{+0.06}_{-0.03}$	$-0.340^{+0.004}_{-0.002}$
$T^{(1)}_{\pi\Sigma}$	3.23	$2.31^{+0.37}_{-0.38}$	$-0.55^{+0.06}_{-0.03}$	4.99 ± 0.38	0.36 ± 0.03
$T^{(0)}_{\pi\Sigma}$	6.45	$-6.09^{+0.56}_{-0.57}$	$-0.41^{+0.12}_{-0.06}$	-0.05 ± 0.57	-0.004 ± 0.041
$T^{(3/2)}_{\pi\Xi}$	-1.61	0.46	-1.23	-2.38	-0.17
$T^{(1/2)}_{\pi\Xi}$	3.23	0.46	-0.48	3.21	0.23
$T_{\pi\Lambda}$	0	0.50 ± 0.06	-1.52	-1.02 ± 0.06	-0.072 ± 0.004

Table 2. Kaon–baryon threshold T-matrices order by order with only octet contributions in unit of fm

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T^{(1)}_{KN}$	-7.63	-8.81	10.11	-6.33	-0.33 (input)
$T^{(0)}_{KN}$	0	16.53	-16.15	0.38	0.02 (input)
$T^{(1)}_{\overline{K}N}$	3.81	3.86	$-0.58 + 6.95i$	$7.09 + 6.95i$	$0.37 + 0.36i$ (input)
$T^{(0)}_{\overline{K}N}$	11.44	-21.48	$-22.56 + 4.17i$	$-32.60 + 4.17i$	$-1.70 + 0.22i$ (input)
$T^{(3/2)}_{K\Sigma}$	-3.81	$5.01^{+0.15}_{-0.36}$	$0.75^{+1.27}_{-0.60} + 2.78i$	$1.95^{+1.28}_{-0.70} + 2.78i$	$0.11^{+0.07}_{-0.04} + 0.16i$
$T^{(1/2)}_{K\Sigma}$	7.63	$3.29^{+0.08}_{-0.18}$	$1.62^{+0.63}_{-0.30} + 0.69i$	$12.54^{+0.64}_{-0.35} + 0.69i$	$0.71^{+0.04}_{-0.02} + 0.04i$
$T^{(3/2)}_{\overline{K}\Sigma}$	-3.81	3.86	$-4.39 + 2.78i$	$-4.34 + 2.78i$	$-0.24 + 0.16i$
$T^{(1/2)}_{\overline{K}\Sigma}$	7.63	$5.58^{+0.23}_{-0.54}$	$6.62^{+1.90}_{-0.91} + 0.69i$	$19.83^{+1.92}_{-1.05} + 0.69i$	$1.12^{+0.11}_{-0.06} + 0.04i$
$T^{(1)}_{K\Xi}$	3.81	$5.01^{+0.15}_{-0.36}$	$1.76^{+1.27}_{-0.60} + 6.95i$	$10.58^{+1.28}_{-0.70} + 6.95i$	$0.61^{+0.07}_{-0.04} + 0.40i$
$T^{(0)}_{K\Xi}$	11.44	$-22.62^{+0.15}_{-0.36}$	$-23.46^{+2.84}_{-1.35} + 4.17i$	$-34.64^{+2.84}_{-1.40} + 4.17i$	$-2.01^{+0.16}_{-0.08} + 0.24i$
$T^{(1)}_{\overline{K}\Xi}$	-7.63	-8.81	$10.83^{+1.79}_{-0.85}$	$-5.61^{+1.79}_{-0.85}$	$-0.32^{+0.10}_{-0.05}$
$T^{(0)}_{\overline{K}\Xi}$	0	$18.82^{+0.31}_{-0.72}$	$-5.14^{+1.79}_{-0.85}$	$13.68^{+1.82}_{-1.12}$	$0.79^{+0.11}_{-0.06}$
$T_{K\Lambda}$	0	$4.98^{+1.04}_{-1.08}$	$-2.81^{+0.63}_{-0.30} + 6.25i$	$2.17^{+1.22}_{-1.12} + 6.25i$	$0.12^{+0.07}_{-0.06} + 0.34i$
$T_{\overline{K}\Lambda}$	0	$4.98^{+1.04}_{-1.08}$	$-1.05^{+0.63}_{-0.30} + 6.25i$	$3.93^{+1.22}_{-1.12} + 6.25i$	$0.22^{+0.07}_{-0.06} + 0.34i$

Table 3. Eta–baryon threshold T-matrices order by order with only octet contributions in unit of fm

	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T_{\eta N}$	$1.29^{+1.32}_{-1.33}$	$2.06 + 8.32i$	$3.35^{+1.32}_{-1.33} + 8.32i$	$(0.17 \pm 0.07) + 0.42i$
$T_{\eta\Sigma}$	$5.49^{+0.67}_{-0.68}$	$1.88 + 5.55i$	$7.37^{+0.67}_{-0.68} + 5.55i$	$(0.40 \pm 0.04) + 0.30i$
$T_{\eta\Xi}$	$11.28^{+1.35}_{-1.46}$	$0.53 + 8.32i$	$11.80^{+1.35}_{-1.46} + 8.32i$	$(0.66 \pm 0.08) + 0.47i$
$T_{\eta\Lambda}$	$-29.41^{+1.99}_{-2.04}$	$2.67 + 16.64i$	$-26.73^{+1.99}_{-2.04} + 16.64i$	$(-1.43 \pm 0.11) + 0.89i$

Table 4. Pion–baryon threshold T-matrices order by order, including decuplet contributions in unit of fm

	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T^+_{\pi N}$	0	$0.33^{+0.02}_{-0.04}$	-0.33	$-0.002^{+0.018}_{-0.043}$	$-0.00014^{+0.00127}_{-0.00297}$ (input)
$T^-_{\pi N}$	1.61	0	$0.19^{+0.04}_{-0.02}$	$1.81^{+0.04}_{-0.02}$	$0.125^{+0.003}_{-0.001}$ (input)
$T^{(3/2)}_{\pi N}$	-1.61	$0.33^{+0.02}_{-0.04}$	$-0.53^{+0.04}_{-0.02}$	-1.81 ± 0.05	-0.125 ± 0.003
$T^{(1/2)}_{\pi N}$	3.23	$0.33^{+0.02}_{-0.04}$	$0.05^{+0.09}_{-0.04}$	$3.61^{+0.18}_{-0.12}$	0.25 ± 0.01
$T^{(2)}_{\pi \Sigma}$	-3.23	-1.05	$0.60^{+0.06}_{-0.03}$	$-4.88^{+0.06}_{-0.03}$	$-0.35^{+0.004}_{-0.002}$
$T^{(1)}_{\pi \Sigma}$	3.23	$-0.21^{+0.37}_{-0.38}$	$-0.79^{+0.06}_{-0.03}$	$2.23^{+0.37}_{-0.38}$	0.16 ± 0.03
$T^{(0)}_{\pi \Sigma}$	6.45	$-2.31^{+0.56}_{-0.57}$	$-0.31^{+0.12}_{-0.06}$	3.82 ± 0.57	0.27 ± 0.04
$T^{(3/2)}_{\pi \Xi}$	-1.61	0.46	-1.34	-2.49	-0.18
$T^{(1/2)}_{\pi \Xi}$	3.23	0.46	-0.58	3.11	0.22
$T_{\pi \Lambda}$	0	0.74 ± 0.06	-1.32	-0.58 ± 0.06	-0.041 ± 0.004

Table 5. Kaon–baryon threshold T-matrices order by order, including decuplet contributions in unit of fm

	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T^{(1)}_{KN}$	-7.63	-8.81	10.11	-6.33	-0.33 (input)
$T^{(0)}_{KN}$	0	16.53	-16.15	0.38	0.02 (input)
$T^{(1)}_{\overline{K}N}$	3.81	3.86	$-0.58 + 6.95i$	$7.09 + 6.95i$	$0.37 + 0.36i$(input)
$T^{(0)}_{\overline{K}N}$	11.44	-21.48	$-22.56 + 4.17i$	$-32.60 + 4.17i$	$-1.70 + 0.22i$ (input)
$T^{(3/2)}_{K\Sigma}$	-3.81	$2.77^{+0.15}_{-0.36}$	$3.90^{+1.27}_{-0.60} + 2.78i$	$2.86^{+1.28}_{-0.70} + 2.78i$	$0.16^{+0.07}_{-0.04} + 0.16i$
$T^{(1/2)}_{K\Sigma}$	7.63	$4.41^{+0.08}_{-0.18}$	$1.08^{+0.63}_{-0.30} + 0.69i$	$13.11^{+0.64}_{-0.35} + 0.69i$	$0.74^{+0.04}_{-0.02} + 0.04i$
$T^{(3/2)}_{\overline{K}\Sigma}$	-3.81	3.86	$-3.70 + 2.78i$	$-3.65 + 2.78i$	$-0.21 + 0.16i$
$T^{(1/2)}_{\overline{K}\Sigma}$	7.63	$2.23^{+0.23}_{-0.54}$	$11.00^{+1.90}_{-0.91} + 0.69i$	$20.86^{+1.92}_{-1.05} + 0.69i$	$1.17^{+0.11}_{-0.06} + 0.04i$
$T^{(1)}_{K\Xi}$	3.81	$2.77^{+0.15}_{-0.36}$	$4.91^{+1.27}_{-0.60} + 6.95i$	$11.49^{+1.28}_{-0.70} + 6.95i$	$0.67^{+0.07}_{-0.04} + 0.40i$
$T^{(0)}_{K\Xi}$	11.44	$-20.39^{+0.15}_{-0.36}$	$-25.23^{+2.84}_{-1.35} + 4.17i$	$-34.18^{+2.84}_{-1.40} + 4.17i$	$-1.98^{+0.16}_{-0.08} + 0.24i$
$T^{(1)}_{\overline{K}\Xi}$	-7.63	-8.81	$11.52^{+1.79}_{-0.85}$	$-4.92^{+1.79}_{-0.85}$	$-0.28^{+0.10}_{-0.05}$
$T^{(0)}_{\overline{K}\Xi}$	0	$14.35^{+0.31}_{-0.72}$	$0.47^{+1.79}_{-0.85}$	$14.82^{+1.82}_{-1.12}$	$0.86^{+0.11}_{-0.06}$
$T_{K\Lambda}$	0	$-2.42^{+1.04}_{-1.08}$	$-2.81^{+0.63}_{-0.30} + 6.25i$	$-5.23^{+1.22}_{-1.12} + 6.25i$	$-0.29^{+0.07}_{-0.06} + 0.34i$
$T_{\overline{K}\Lambda}$	0	$-2.42^{+1.04}_{-1.08}$	$-1.05^{+0.63}_{-0.30} + 6.25i$	$-3.47^{+1.22}_{-1.12} + 6.25i$	$-0.19^{+0.07}_{-0.06} + 0.34i$

Table 6. Eta–baryon threshold T-matrices order by order, including decuplet contributions in unit of fm

	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T_{\eta N}$	$-9.67^{+1.32}_{-1.33}$	$3.81 + 8.32i$	$-5.87^{+1.32}_{-1.33} + 8.32i$	$(-0.29 \pm 0.07) + 0.42i$
$T_{\eta \Sigma}$	$9.75^{+0.67}_{-0.68}$	$3.10 + 5.55i$	$12.86^{+0.67}_{-0.68} + 5.55i$	$(0.70 \pm 0.04) + 0.30i$
$T_{\eta \Xi}$	$1.63^{+1.35}_{-1.46}$	$1.12 + 8.32i$	$2.75^{+1.35}_{-1.46} + 8.32i$	$(0.15 \pm 0.08) + 0.47i$
$T_{\eta \Lambda}$	$-17.60^{+1.99}_{-2.04}$	$6.33 + 16.64i$	$-11.27^{+1.99}_{-2.04} + 16.64i$	$(-0.60 \pm 0.11) + 0.89i$

Table 7. Pion–baryon threshold T-matrices order by order in large N_c limit in unit of fm

	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T^+_{\pi N}$	0	$0.38^{+0.02}_{-0.04}$	-0.38	$-0.002^{+0.018}_{-0.043}$	$-0.00014^{+0.00127}_{-0.00297}$ (input)
$T^-_{\pi N}$	1.61	0	$0.19^{+0.04}_{-0.02}$	$1.81^{+0.04}_{-0.02}$	$0.125^{+0.003}_{-0.001}$ (input)
$T^{(3/2)}_{\pi N}$	-1.61	$0.38^{+0.02}_{-0.04}$	$-0.57^{+0.04}_{-0.02}$	-1.81 ± 0.05	-0.125 ± 0.003
$T^{(1/2)}_{\pi N}$	3.23	$0.38^{+0.02}_{-0.04}$	$-0.01^{+0.09}_{-0.04}$	$3.61^{+0.18}_{-0.12}$	0.25 ± 0.01
$T^{(2)}_{\pi\Sigma}$	-3.23	-1.05	$-0.66^{+0.06}_{-0.03}$	$-4.94^{+0.06}_{-0.03}$	$-0.35^{+0.004}_{-0.002}$
$T^{(1)}_{\pi\Sigma}$	3.23	$1.87^{+0.37}_{-0.38}$	$-0.82^{+0.06}_{-0.03}$	4.28 ± 0.38	0.30 ± 0.03
$T^{(0)}_{\pi\Sigma}$	6.45	$-5.44^{+0.56}_{-0.57}$	$-0.41^{+0.12}_{-0.06}$	0.61 ± 0.57	0.04 ± 0.04
$T^{(3/2)}_{\pi\Xi}$	-1.61	0.46	-1.39	-2.55	-0.18
$T^{(1/2)}_{\pi\Xi}$	3.23	0.46	-0.64	3.05	0.22
$T_{\pi\Lambda}$	0	0.42 ± 0.06	-1.35	-0.93 ± 0.06	-0.066 ± 0.004

Table 8. Kaon–baryon threshold T-matrices order by order in large N_c limit in unit of fm

	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T^{(1)}_{\mathrm{K}N}$	-7.63	-8.81	10.11	-6.33	-0.33 (input)
$T^{(0)}_{\mathrm{K}N}$	0	16.53	-16.15	0.38	0.02 (input)
$T^{(1)}_{\overline{\mathrm{K}}N}$	3.81	3.86	$-0.58+6.95\mathrm{i}$	$7.09+6.95\mathrm{i}$	$0.37+0.36\mathrm{i}$ (input)
$T^{(0)}_{\overline{\mathrm{K}}N}$	11.44	-21.48	$-22.56+4.17\mathrm{i}$	$-32.60+4.17\mathrm{i}$	$-1.70+0.22\mathrm{i}$ (input)
$T^{(3/2)}_{\mathrm{K}\Sigma}$	-3.81	$3.17^{+0.15}_{-0.36}$	$4.62^{+1.27}_{-0.60}+2.78\mathrm{i}$	$3.98^{+1.28}_{-0.70}+2.78\mathrm{i}$	$0.22^{+0.07}_{-0.04}+0.16\mathrm{i}$
$T^{(1/2)}_{\mathrm{K}\Sigma}$	7.63	$4.21^{+0.08}_{-0.18}$	$1.11^{+0.63}_{-0.30}+0.69\mathrm{i}$	$12.95^{+0.64}_{-0.35}+0.69\mathrm{i}$	$0.73^{+0.04}_{-0.02}+0.04\mathrm{i}$
$T^{(3/2)}_{\overline{\mathrm{K}}\Sigma}$	-3.81	3.86	$-3.44+2.78\mathrm{i}$	$-3.39+2.78\mathrm{i}$	$-0.19+0.16\mathrm{i}$
$T^{(1/2)}_{\overline{\mathrm{K}}\Sigma}$	7.63	$2.82^{+0.23}_{-0.54}$	$11.95^{+1.90}_{-0.91}+0.69\mathrm{i}$	$20.40^{+1.92}_{-1.05}+0.69\mathrm{i}$	$1.26^{+0.11}_{-0.06}+0.04\mathrm{i}$
$T^{(1)}_{\mathrm{K}\Xi}$	3.81	$3.17^{+0.15}_{-0.36}$	$5.63^{+1.27}_{-0.60}+6.95\mathrm{i}$	$12.61^{+1.28}_{-0.70}+6.95\mathrm{i}$	$0.73^{+0.07}_{-0.04}+0.40\mathrm{i}$
$T^{(0)}_{\mathrm{K}\Xi}$	11.44	$-20.79^{+0.15}_{-0.36}$	$-25.43^{+2.84}_{-1.35}+4.17\mathrm{i}$	$-34.77^{+2.84}_{-1.40}+4.17\mathrm{i}$	$-2.01^{+0.16}_{-0.08}+0.24\mathrm{i}$
$T^{(1)}_{\overline{\mathrm{K}}\Xi}$	-7.63	-8.81	$11.78^{+1.79}_{-0.85}$	$-4.65^{+1.79}_{-0.85}$	$-0.27^{+0.10}_{-0.05}$
$T^{(0)}_{\overline{\mathrm{K}}\Xi}$	0	$15.15^{+0.31}_{-0.72}$	$1.65^{+1.79}_{-0.85}$	$16.79^{+1.82}_{-1.12}$	$0.97^{+0.11}_{-0.06}$
$T_{\mathrm{K}\Lambda}$	0	$3.45^{+1.04}_{-1.08}$	$-2.81^{+0.63}_{-0.30}+6.25\mathrm{i}$	$0.64^{+1.22}_{-1.12}+6.25\mathrm{i}$	$0.04^{+0.07}_{-0.06}+0.34\mathrm{i}$
$T_{\overline{\mathrm{K}}\Lambda}$	0	$3.45^{+1.04}_{-1.08}$	$-1.05^{+0.63}_{-0.30}+6.25\mathrm{i}$	$2.40^{+1.22}_{-1.12}+6.25\mathrm{i}$	$0.13^{+0.07}_{-0.06}+0.34\mathrm{i}$

Table 9. Eta–baryon threshold T-matrices order by order in large N_c limit in unit of fm

	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	Total	Scattering lengths
$T_{\eta N}$	$0.51^{+1.32}_{-1.33}$	$4.30+8.32\mathrm{i}$	$4.80^{+1.32}_{-1.33}+8.32\mathrm{i}$	$(0.24\pm0.07)+0.42\mathrm{i}$
$T_{\eta\Sigma}$	$4.70^{+0.67}_{-0.68}$	$3.38+5.55\mathrm{i}$	$8.08^{+0.67}_{-0.68}+5.55\mathrm{i}$	$(0.44\pm0.04)+0.30\mathrm{i}$
$T_{\eta\Xi}$	$8.14^{+1.35}_{-1.46}$	$1.23+8.32\mathrm{i}$	$9.38^{+1.35}_{-1.46}+8.32\mathrm{i}$	$(0.53\pm0.08)+0.47\mathrm{i}$
$T_{\eta\Lambda}$	$-27.06^{+1.99}_{-2.04}$	$7.28+16.64\mathrm{i}$	$-19.78^{+1.99}_{-2.04}+16.64\mathrm{i}$	$(-1.06\pm0.11)+0.89\mathrm{i}$

scattering lengths in three cases by including the counterterms. We found that chiral expansion in several channels converges well without considering the decuplet contributions. When decuplet contributions were considered, the convergence of the chiral expansion does not change significantly. Our calculation indicates that $a_{\eta\Lambda}$ is negative. Whether η-mesic hypernuclei is possible requires further investigations. We expect the numerical results are useful to model constructions for meson–baryon interaction.

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